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Adsorption of Piecewise Directed Random Walks on Sierpinski Fractals

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Abstract

Studies of the random walk problem on fractal lattices with adsorbing boundaries are important for practical (as a model for polymer adsorption on rigid impenetrable surfaces, when the polymer–solvent system is in a nonhomogenous–fractal container), as well as in the wider context of the surface critical phenomena physics. In this work we study the problem of adsorption of piecewise directed random walks (PDW) on fractal lattices that belong to the Sierpinski–gasket (SG) family. By applying the renormalization group (RG) method in real space we calculate exactly critical exponent Φ, associated with the number of the adsorbed steps, for the first 1000 members of the SG family. The exact results are numerically analyzed, the possible relation between the exponent Φ and the fractal properties is discussed, as well as the asymptotic behavior of the exponent Φ in the vicinity of the fractal–homogeneous lattice crossover, when PDW on SG fractal becomes the common directed walk on the triangular lattice. In addition, we compare our results with the exact and Monte Carlo RG results for the self-avoiding walk model on SG fractals, and, finally we test the Bouchaud–Vannimenus bounds for Φ.

1. Introduction

Under certain conditions the polymer chain in a good solvent in the vicinity of an attractive impenetrable wall can undergo an adsorption–desorption transition, which is a problem of great theoretical and practical importance. The asymptotic behavior of the average number M of adsorbed monomers can be summarized as:

$$M \sim \begin{cases} N(T_a - T)^{1/\Phi - 1}, & \text{for } T < T_a\,, \\ N^{\Phi}, & \text{for } T = T_a\,, \\ (T - T_a)^{-1}, & \text{for } T > T_a\,, \end{cases} \tag{1}$$

where N denotes the average total number of monomers, T is the temperature, T_a is the critical temperature of the adsorption, and Φ is the crossover exponent [1]. The self-avoiding random walk (SAW) model on lattices, with an increased probability of making steps along the lattice boundary (adsorbing wall), has been accepted as a good model for this phenomenon. In this report we present the problem of the adsorption of a special nontrivial type of SAW, the so-called piecewise directed random walk (PDW), on the Sierpinski gasket (SG) family of fractals, which allows us to calculate exactly the exponent Φ on any SG fractal (which is numerical analysis, function $g(b)$ in Eq.7 does not tend to 0 faster than $1/\ln b$, which implies that

$$\Phi_{PDW} \sim \frac{\ln K}{\ln b} \tag{8}$$

for $b \to \infty$. In other words, although we can not say that Φ_{PDW} is some simple function of the fractal and spectral dimensions of the corresponding fractal (we could not find any, at least), we can say that behavior of Φ_{PDW} in the vicinity of the fractal–Euclidean lattice crossover is governed by the fractal dimension. Again, this is similar to the ν_{PDW} behavior [3]. Finally, one can also notice in Fig.2 that our values of Φ_{PDW} satisfy the Bouchaud–Vannimenus upper $\Phi_u = 1 - (\bar{d} - d_S)\nu$ and lower $\Phi_l = \bar{d}/d_S$ bounds (with d_S being the fractal dimension of the adsorbing wall, i.e. $d_S = 1$ in our case), proposed for the SAW on fractals, in contrast to the SAW case, where the lower bound, according to MCRG results [6], is violated for $b \geq 12$. It is interesting to note here that for large values of b lower bound $\Phi_l \sim 1/\ln b$ behaves as Φ_{PDW} (Eq.8). Similar conclusions were recently obtained for the simple random walk on n–simplex lattices [9] and for the SAW on the same class of lattices in the $n \to \infty$ limit [10].

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